Find the radius (and interval) of convergence for

1. Recall the geometric series

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \quad |r| < 1$$

Let $a = 1$ and $r = x$

(a) Write the sum of the series:

$$\frac{a}{1-r} = \frac{1}{1-x}$$

(b) How would this be related to $f(x) = \frac{1}{1-x}$?

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} r^n = \quad + r + r^2 + \cdots + r^n + \cdots$$

(c) What is the interval of convergence?

Root Test

$$\lim_{n\to\infty} |ax^n|^{1/n} = \lim_{n\to\infty} |x| = x < 1,$$ Interval $(1,1)$

(d) Adapt the series so it is centered at -1 → $(x-1)$ we want $x-1$ to match 1

$$\frac{1}{1-x} = \frac{1}{2-(x+1)} = \frac{\frac{1}{2}}{1-\frac{1}{2}(x+1)}$$ so $a = \frac{1}{2}$ $r = \frac{x+1}{2}$

So

$$\sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{x+1}{2}\right)^n = \frac{1}{1-x}$$

Converges

$$\boxed{\begin{gathered}\sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{x+1}{2}\right)^n = \frac{1}{2}\sum ar^n \text{ if } a=1,\ r=\frac{x+1}{2} \\ -1 < \left|\frac{x+1}{2}\right| < 1 \\ -2 < x+1 < 2 \\ -3 < x < 1\end{gathered}}$$

$1 - r$

with $x+1 = r$

$$\frac{a}{1-r} \Rightarrow \frac{a}{1+1-(x+1)}$$

Verify w/ Ratio

$$\lim_{n\to\infty} \left| \frac{\frac{1}{2}\left(\frac{x+1}{2}\right)^{n+1}}{\frac{1}{2}\left(\frac{x+1}{2}\right)^n} \right| < 1$$

$$\lim_{n\to\infty} \left|\frac{x+1}{2}\right| < 1$$